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Curvature as Obstacle to a Photo-Resistor Sensor of Illumining and Their Minimal Sensing Region Part II: Their Transitory Analysis and the Non-Dimensional λ

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Abstract: In this study are analyzed from point of view of laboratory and test-rehearsals the functioning of an illuminating sensor considering the fact that the measures and sensing must be realized to start of the response signal to the luminous efficiency described for their mean foreseen behavior given for the corresponding integral equation to their efficiency. Here are considered the efficiency function and the response signal of the sensor. The integral equation represents the functioning of the sensor submitted to a luminous efficiency λ, which will be relevant to the detection and measure of the illuminating curvature energy. Also are obtained images of spectra bandwidth of the mean curvature spectra and the dimensionless value λ. In this last point are established two important results, one theorem and one lemma in signal and systems analysis applied to the efficacies and efficiency of the illuminating sensor considering the energy spectra of the curvature, the luminous energy, and the illuminating energy density. Likewise, is determined the curvature energy as the first order derivative of illuminating energy density divided for the electric charge used in the photo-resistive component of the sensor. Also are obtained 2-dimensional geometrical models or behavior surfaces of curvature energy, efficiency and their efficacies accord with the laboratory results.

Keywords: Curvature Energy, Illuminating Sensor, Luminous Efficiency, Luminous Intensity, Sensing Obstacle Problem

1. Introduction

As has been studied, the illumination is the action of illuminating field $\aleph$, whose illuminating flow that impact in a surface determines differentiated regions of clear and dark illuminating fields which establish a measure of the illuminating gradient as phenomena of a space affected by a light source [1].

Then the boundary between both regions is not a well-delimitated line and their energy require a weak topology [2] defined by a norm or length $\|u\|_{s,p}$. This was observed from a point of experimental view using the gradients of illuminating field defining the condition between illuminating (clear and dark) fields, as the limit condition:

$$\lim_{u\to\upsilon}\nabla(\upsilon-u)=\lim_{u\to\upsilon}\aleph^{-}-\aleph^{+}=\delta\aleph., \tag{1}$$

But from a point of electronic view what is happening with a sensor immerse in an illuminating field, which detects and measure the illuminating gradient in a geometrical enclosure through their mean curvature energy?

Well, the illuminating field is re-interpreted as a Jacobi field, where the mean curvature energy stays in the domain $|H(\omega_1,\omega_2)|\leq\kappa$. We have considered the response signal of the illuminating sensor with capacitance characterized for $1/R(\varphi)$, to the co-cycles of curvature energy [3]:

$$h(t)=V_{in}e^{-\left(\frac{\lambda}{R(\varphi)}\right)t}, \tag{2}$$

which to our illuminating sensor we have identified the boundary conditions given for:

$$\lim_{\varphi\to 0}R(\varphi)=\infty, \tag{3}$$

$$\lim_{R\to\infty}\left[-\frac{\lambda}{R(\varphi)}\right]=0, \tag{4}$$

But considering the boundary condition (3), we have in equivalent way that $h(0)=V_{in}$, and to the condition (4), we have $h(\infty)=0$.

2. Dynamics Analysis

2.1. The Response Signal as Response to Little Voltage Variations: The Curvature Energy Is Re-interpreted as Variations of Luminous Energy Density

The response signal of voltage is the law of the process [1]

$$R(\varphi)\frac{dV}{dt}=-\lambda(V_{in}-V_C), \tag{5}$$

To the behavior of the sensor considering a complete transitory analysis of system along a time interval in the experimentation [4], we need to take the bordering conditions (3)-(4) adding the luminous efficacy interact with the response signal of voltage. Likewise, this interaction can stay established by the convolution:

$$(h*\lambda)(t)=\int_0^T h(t-\tau)\lambda(\tau)d\tau, \tag{6}$$

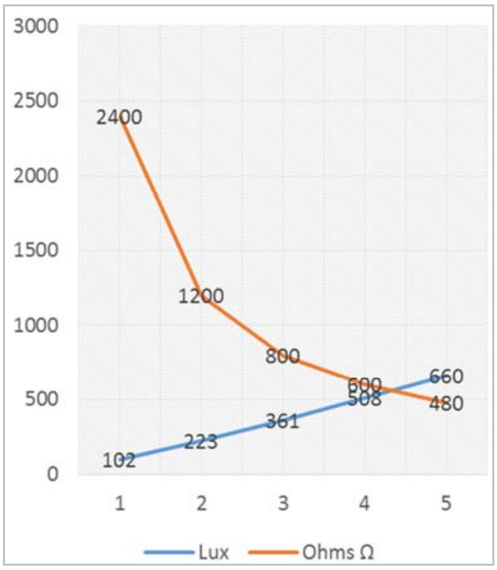


Figure 1. *Illumining intensity versus resistance. See the TABLE 1, of [1].*

Table 1. *Illumining intensity versus resistance [1].*

Lux	Ohms Ω
102	2400 Ω
223	1200 Ω
361	800 Ω
508	600 Ω
660	480 Ω
818	400 Ω
980	343 Ω

However, we want the illuminating sensing considering the input voltage, which is defined for the initial condition $h(0)=V_{in}$, and the remainder of the resistance due the luminosity perceived by the sensor, which is defined by the perceived and thus flux luminous efficiency [4] $\lambda(t)$.

Also $R(\varphi)$, is the value of the flux detected for the resistance, which is found in the intersection of the graph of the Figure 1. But this intersection satisfies from the argument of the exponential in the response signal that

$$\lambda=\frac{\Phi}{R(\varphi)}=\frac{\Phi}{kR_\varphi}, \tag{7}$$

which is a non-dimensional. In the case of the intersection, $\lambda=1$. This corresponds to their normalization. However, what happens when is not normalized?

We have a coefficient, which vary depending of the illumining efficacy [1]. Then we can to the general case a function $\lambda(t)$, to time sufficiently large of sensor functioning. Likewise, and consider the convolution (6), the behavior of our photo-resistance due the luminous flux given by (7) and the limit condition (4), we can write de following integral equation that will describe the efficiency of our illuminating sensor in a sufficiently large time period [5, 6]:

$$\eta(t)=h(0)\lambda(t)+\int_0^\infty h(t-\tau)\lambda(\tau)d\tau, \tag{8}$$

The kernel $h(t-\tau)$, obeys to the interaction, for one side of the response signal of system and for other the efficacy of the sensor to realize the sensing process.

2.2. The Behavior of the Luminous Efficacy and Efficiency. Efficiency to Optimize the Illumination

The behavior of the luminous efficacy and efficiency will be obtained solving the integral equation (8). Likewise, applying the Laplace transform to both members of (8) we have:

$$\begin{aligned}\mathrm{H}(p)&=V_{in}\Lambda(p)+\mathcal{L}\{(h*\lambda)(\mathrm{t})\}\\&=V_{in}\Lambda(p)+\mathrm{H}(p)\Lambda(p),\end{aligned} \tag{9}$$

where

$$\Lambda(p)=\frac{\mathrm{H}(p)}{(V_{in}+\mathrm{H}(p))}, \tag{10}$$

1 The illuminating efficacy comes given as: $\lambda=\frac{\Phi}{\Phi_e}=\frac{\int_0^\infty \Lambda(\omega)\Phi_{e,\omega}d\omega}{\int_0^\infty \Phi_{e,\lambda}d\omega}$,

here Φ, is the illuminating flux, Φ_e, is the radiant flux, $\Phi_{e,\lambda}$, is the spectral radiant flux and $\Lambda(\omega)$, is the spectral luminous efficacy. Here $\Lambda(\omega)$, is the spectral density of a function plated to the non-dimensional $\lambda(t)$.

which explicitly considering the signal response of the system with $\lambda = 1$, we have that (10) takes the form:

$$\Lambda(p) = \frac{1}{V_{in}\left(p + \dfrac{R(\varphi)+1}{R(\varphi)}\right)}, \tag{11}$$

which exists in the space $\operatorname{Re} p > -\dfrac{R(\varphi)+1}{R(\varphi)}$.

Then applying $\mathcal{L}^{-1}$, to (11) we find that:

$$\lambda(t) = \frac{1}{V_{in}} e^{-\left(\frac{R(\varphi)+1}{R(\varphi)}\right)t} u(t), \tag{12}$$

See the solution (12) to an input voltage (Figure 2). In addition, we have their Heaviside support surface (Figure 3).

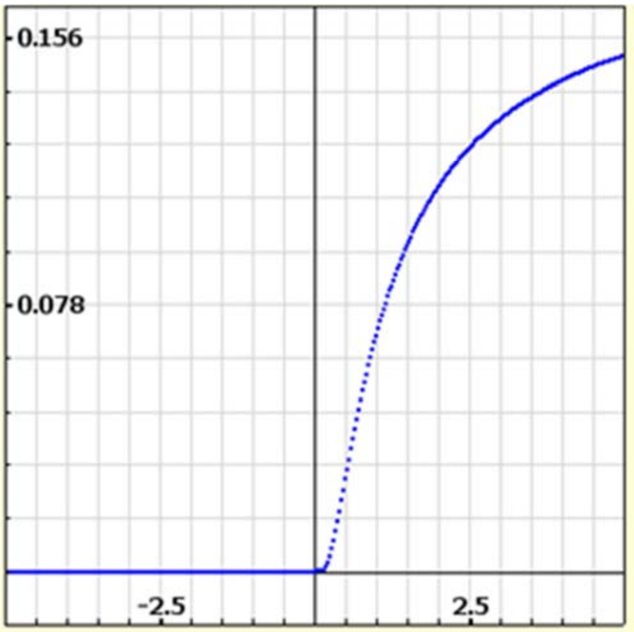


Figure 2. *Efficiency curve of illuminating sensor, considering the input voltage of* $V_{in} = 5Volts$.

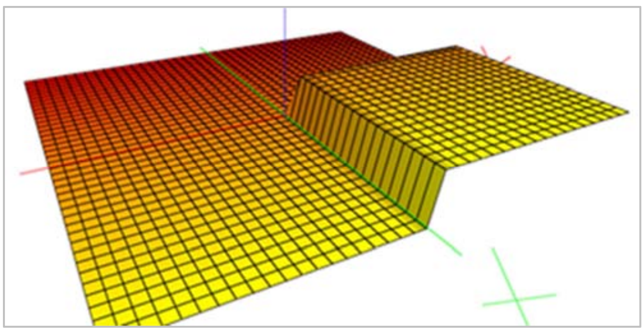


Figure 3. *Heaviside support surface to the solution (12).*

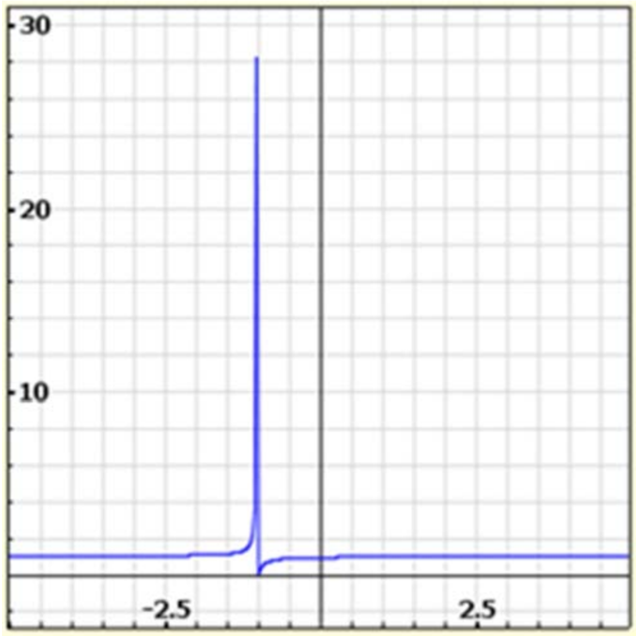


Figure 4. *Example of efficacy* Λ, *of the sensor with following flux and resistance data to a resonance arbitrary:* $\left\{\dfrac{-7000,000}{16(7,000,000)\omega + 7,000,001)}\right\}$.

The tendency to the equilibrium of the response signal stays guarantied in a sufficiently short interval to the efficacy. Indeed, the efficacy to each $t_0 \leq t$, will be the function:

$$\lambda(t) = \frac{1}{2\pi V_{in}} e^{-2t} u(t), \tag{13}$$

which show a behavior tending to the equilibrium or stable in an interval $t_0 \leq t$. Indeed, we consider as initial condition $\lambda_0 = \lambda(t_0)$,

$$\left|\lambda_0 e^{-2(t_0 - t)} u(t) - \bar{\lambda}_0 e^{-2(t_0 - t)} u(t)\right| = e^{-2(t - t_0)} u(t) \left|\lambda_0 - \bar{\lambda}_0\right|,$$

when $t_0 \leq t$, if $\left|\lambda_0 - \bar{\lambda}_0\right| < \varepsilon e^{-2t_0}$, and the limit

$$\lim_{t\to\infty} e^{-2(t-t_0)} u(t) \left|\lambda_0 - \bar{\lambda}_0\right| = 0,$$

Thus, the solution (13) is asymptotic stable in the short interval, which the sensor detect the change of illuminating.

Note that $\lambda_0 = \lambda(t_0) = 1/2\pi V_{in} e^{-2t_0} u(t_0) \to 0$, when $t_0 \to 0$. This can be analyzed in the figures 2 and 5.

2.3. Experiments and Results in Laboratory

The meaning of (12) is the fact of that the response of system goes being more rapid (effective) conforms go decreasing the light scattering, being applied a constant voltage sufficiently large in the time. However, what we have in efficiency?

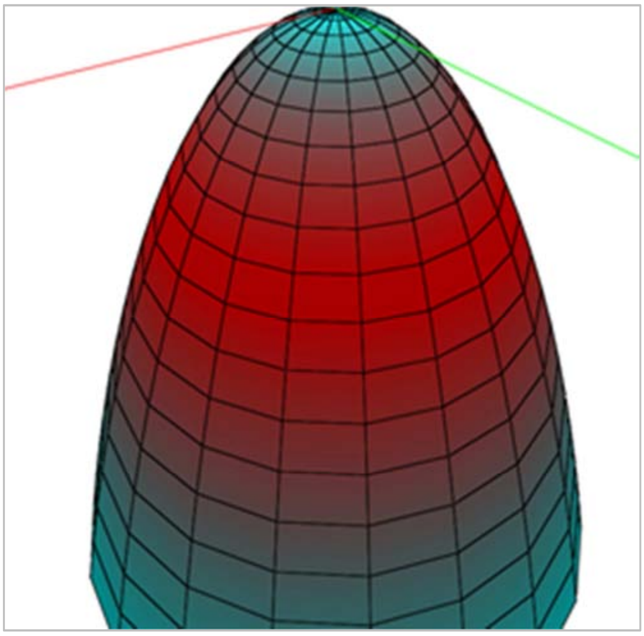
Efficacy not implies efficiency necessarily. To this, we require the geodesic behavior under action of the system response signal.

The efficacy is given in each time as response in frequency (illuminating efficacy). Then this comes given as[2]:

$$\Lambda(\omega)=\frac{R(\varphi)}{V_{in}(R(\varphi)+R(\varphi)j\omega+1)}, \tag{14}$$

See the Figure 4, with some flux and resistance data.

The efficiency is given by the red color region (see the figure 5 A), and B)), which goes being degraded outside of the sensing region. However, their efficacy is related with curvature energy through the variation of the illuminating energy density.



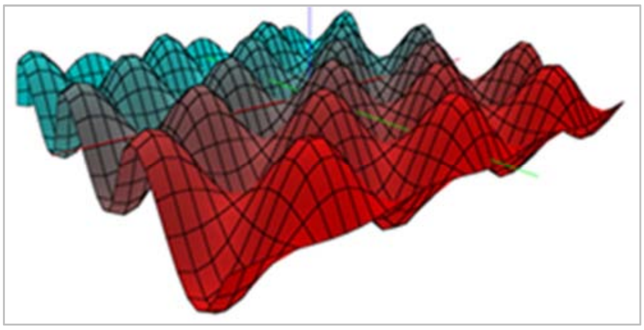


Figure 5. *A). Efficiency given by the curvature energy zone. B). Curvature energy or spectra.*

The variation of the illuminating energy density on the electric charge unit (derived of the voltage) is their curvature energy.

Indeed, we consider the relation between the illuminating flux Φ, and the illuminating energy density ξ, given in [1]. Then is clear that illuminating energy density is the rate of the luminous energy in a volume. Then their curvature energy will be the variation respect to the time of illuminating energy density multiplied for the reciprocal of electric charge obtained for the electrons flow.

Indeed, we consider the following result.

Lemma (F. Bulnes) 2. 3. 1. The curvature energy is the variation of the illuminating energy density on the electric charge [7] obtained

$$\kappa=\frac{1}{Q}\frac{d\xi}{ds}, \tag{15}$$

Proof. We consider the illuminating energy density as:

$$\xi=\frac{\Phi s}{Vol}\left[=\frac{Joule\times\sec}{meter^3}\right], \tag{16}$$

Remember that the illuminating flux unit is, $Lumen = Joule$. For other way, curvature energy is given by

$$\kappa\left[=\frac{1}{Coulumb}\frac{lumen}{meter^3}\right]\left[=\frac{Joule}{Coulumb\times meter^3}\right],$$

If we derivate the luminous energy given (which has a linear behavior) by

$$E_v=\Phi s, \tag{17}$$

We have $\frac{\Phi}{Vol}\left[=\frac{joule}{meter^3}\right]$. Then multiplying this for $1/Q$, we have dimensions of curvature energy. Thus (15) is a change rate of illuminating energy density respect to the time.

It is energy or has energy character, which spectrally is curvature energy [1].

These variations are given as variations of response $G(\omega)=\omega\Xi(\omega)$, where their dimensions are $\frac{1}{\sec}\times\frac{Joule\times\sec}{meter^3}$. The dimensions curvature energy also are obtained multiplying for $1/Q$.

Then the Fourier transform in this case is the identity transformation [5, 8], $\frac{d\xi}{ds}=\int_{-\infty}^{\infty}e^{-j\omega s}\xi'(s)ds$. This prove the proposition.

Then the region (enveloping) where is subjacent the efficiency is the bounded between geodesic curves whose spectra is in the interval $|H(\omega_1,\omega_2)|\leq\kappa$. Indeed, experimentally we have the efficacy to some resonances $\omega_1,\omega_2,\omega_3,\ldots$ (see the Figures 12, 13 and 14 A), B), and C) in the final paper).

We can to enunciate the principle valid in the nature and established in our sensor device:

"The efficiency is the sum of their efficacies"

Then we can give the following result.

Theorem (F. Bulnes) 2. 3. 1. The efficiency of illuminating sensor MLCGD is the following sum

2 This spectral density can be resumed as:

$$\Lambda(\omega)=\frac{1}{V_{in}}\frac{1}{\omega+2}.$$

$$\lambda(t) = \frac{1}{2\pi V_{in}} \sum_{n=0}^{\infty} (-1)^n \frac{(2t)^n}{n!} u(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Lambda(\omega) e^{j\omega t} d\omega, \quad (18)$$

where $\Lambda(\omega)$, is the efficacy to the corresponding resonance ω. The functions $(2t)^n / n!$, are obtained with much approximation in laboratory to the cases $n = 1$, and $n = 2$ (see the figures 1 and figure 13 in the end of the paper).

Proof. We consider the power series of $e^{-2t}u(t)$, which is the inverse Fourier transform $\lambda(t)$.

In the energy signals context all polynomial functions derived of $(2t)^n / n!$, [5] have very little contributions to the efficiency, except to cases $n = 0$, $n = 1$, and $n = 2$. To $n > 2$, the efficacies are less meaning to the behavior of $\lambda(t)$, to different distance between light source and illuminating sensor/LDR (see the Figure 6, Figure 7, Figure 8 and Figure 9).

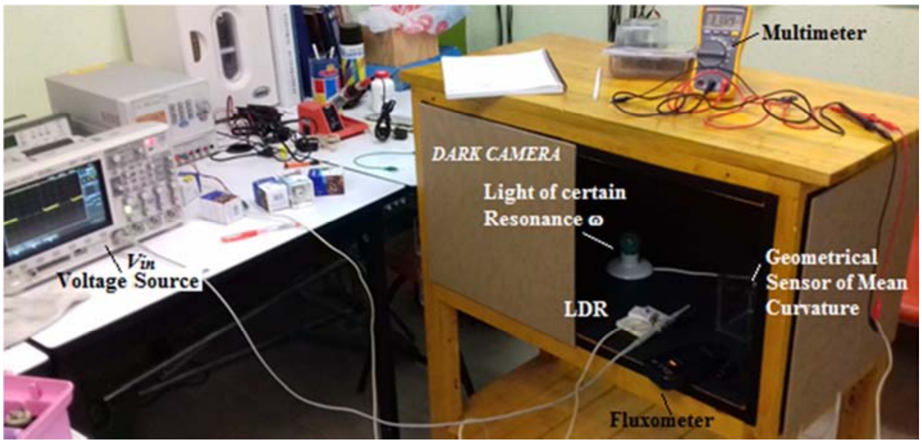


Figure 6. *Evaluation of efficacies and efficiency of the sensor though their transitory analysis and of response variation.*

Figure 7. *Electromechanical component of Geometrical sensor of mean curvature [10, 11].*

However, discrete Fourier series can approximate the efficacies where the Heaviside function is the response voltage along the sensing process.

Likewise, we consider the efficacies measure in direct time of 0, to 2, seconds with increments to second centesimos. In the Fourier transform were obtained the values to the frequencies of $0 \leq \omega \leq 100Hz$, with respects conjugates (see the Figure 8, and Figure 9).

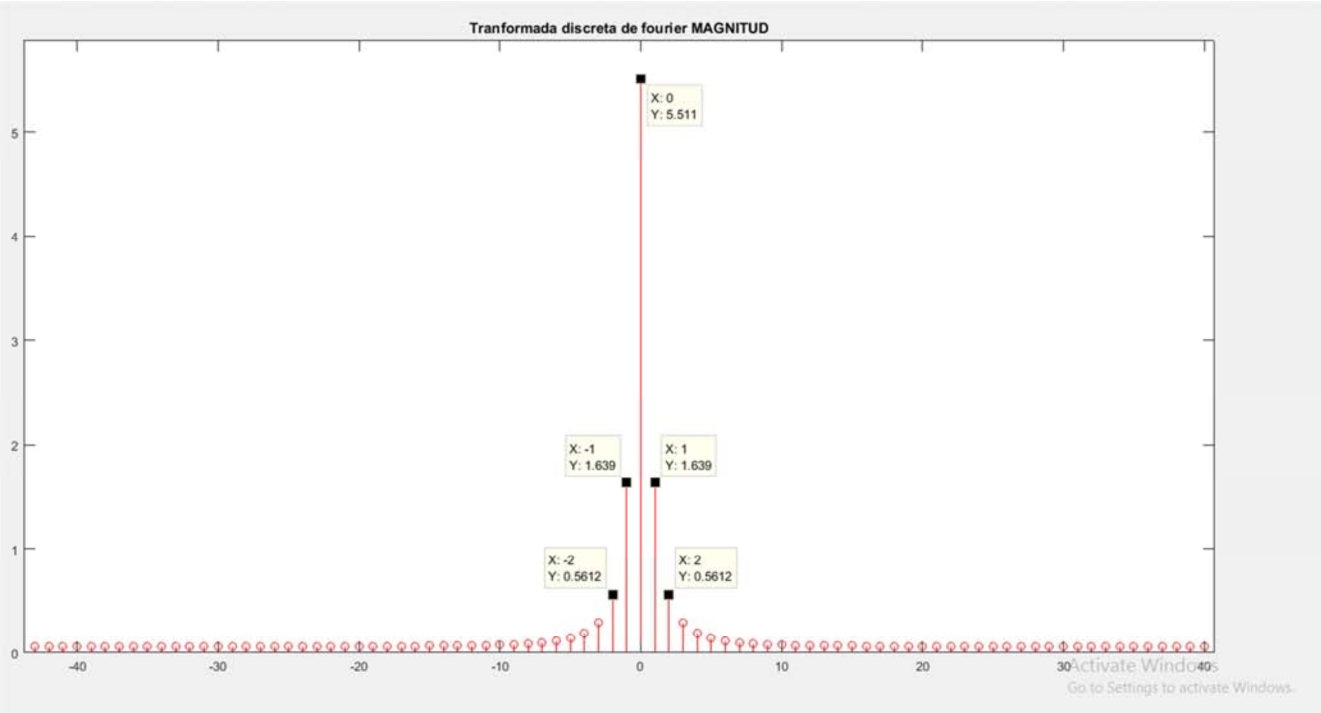


Figure 8. *Discrete Fourier transform: Spectra of magnitude.*

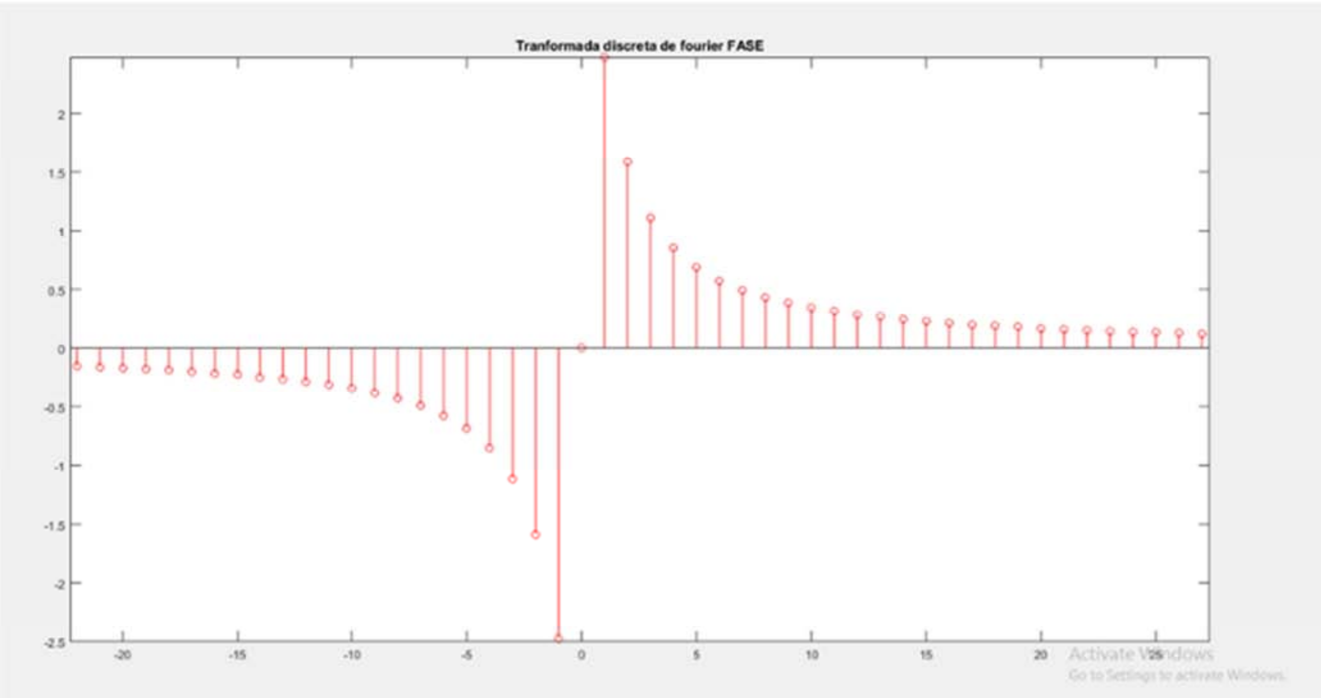


Figure 9. *Discrete Fourier transform: Spectra of phase.*

3. Photonics-Electronics Remarks

The immediate effect in low frequencies generates minor resistance, thus there is more electrical conductivity [7]. The photons in major frequencies have minor collision capacity of effective mode with electrons to carry them to an energy level such that can be became in valence electrons [12-15].

The captor sensor device of mean light has a response of oscillating transitory type and is approximately stabilized in 60 seconds average. This happens when instantaneously is energized the light source and light incidence on said sensor.

Then in function of the light intensity, there is a proportional variation in electrical resistance.

The change of photo-sensible material state during a time, as phenomena, is nearest related with the efficacy that each incident light color.

We can establish a behavior surface of the efficiency due the frequency and resistance to infinite efficacies obtained to different resonances $\omega_1, \omega_2, \omega_3, \ldots$, and the behavior of R, established in the experiments (see the Figure 10). The efficiency always will be positive in a frequency regime.

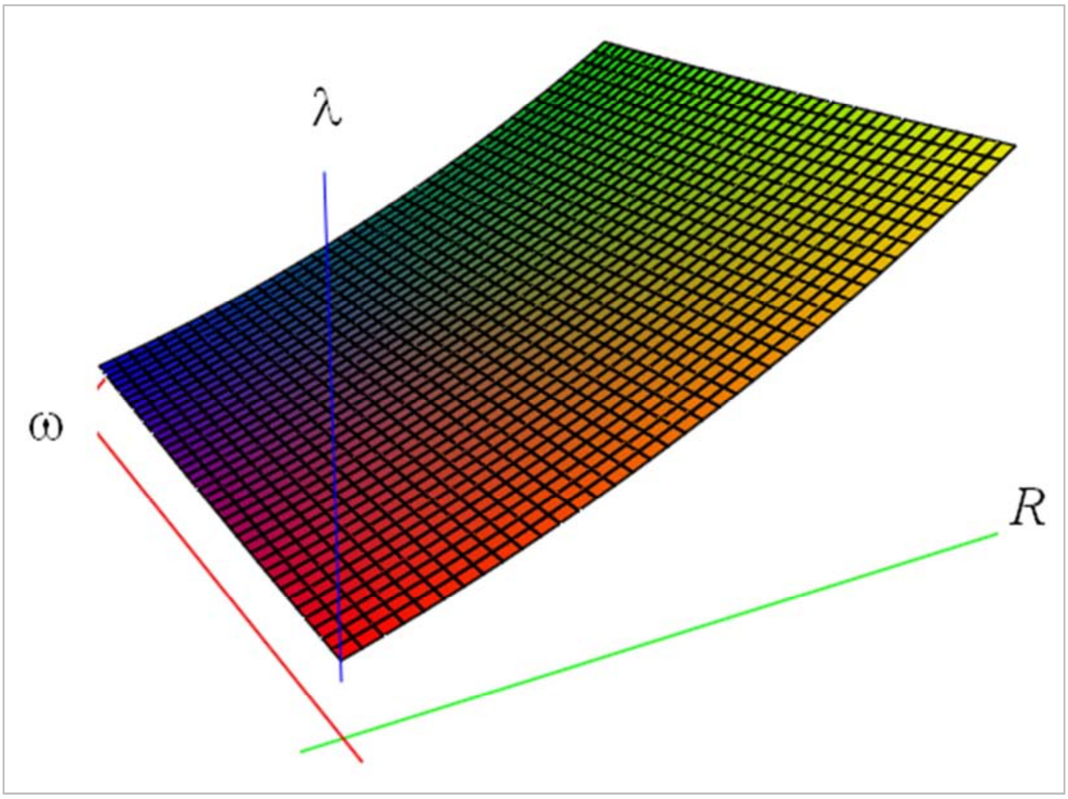


Figure 10. *Andy's" Surface of efficiency* $\lambda = \lambda(R(\varphi), \omega)$, *depending of frequency and resistance. This surface include all curves resistive effect (figures 8) also.*

The efficiency is the infinite sum of efficacies as was proved in the before section and to our sensor this is satisfied.

The curvature energy of the illuminating sensor in functioning under low resistance. This can be viewed through average behavior surface (Figure 11).

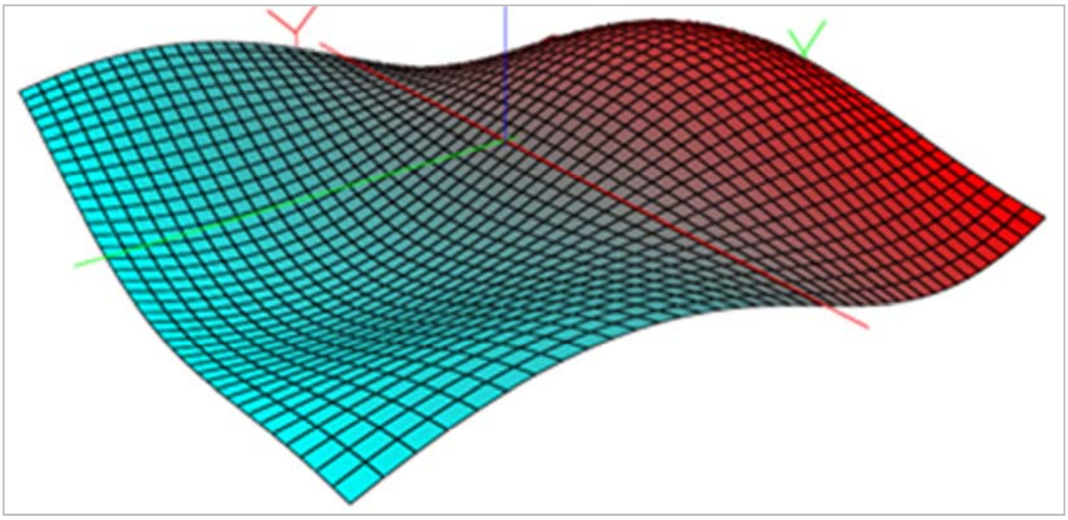


Figure 11. *The curvature energy [9] surface to low resistance of the sensor.*

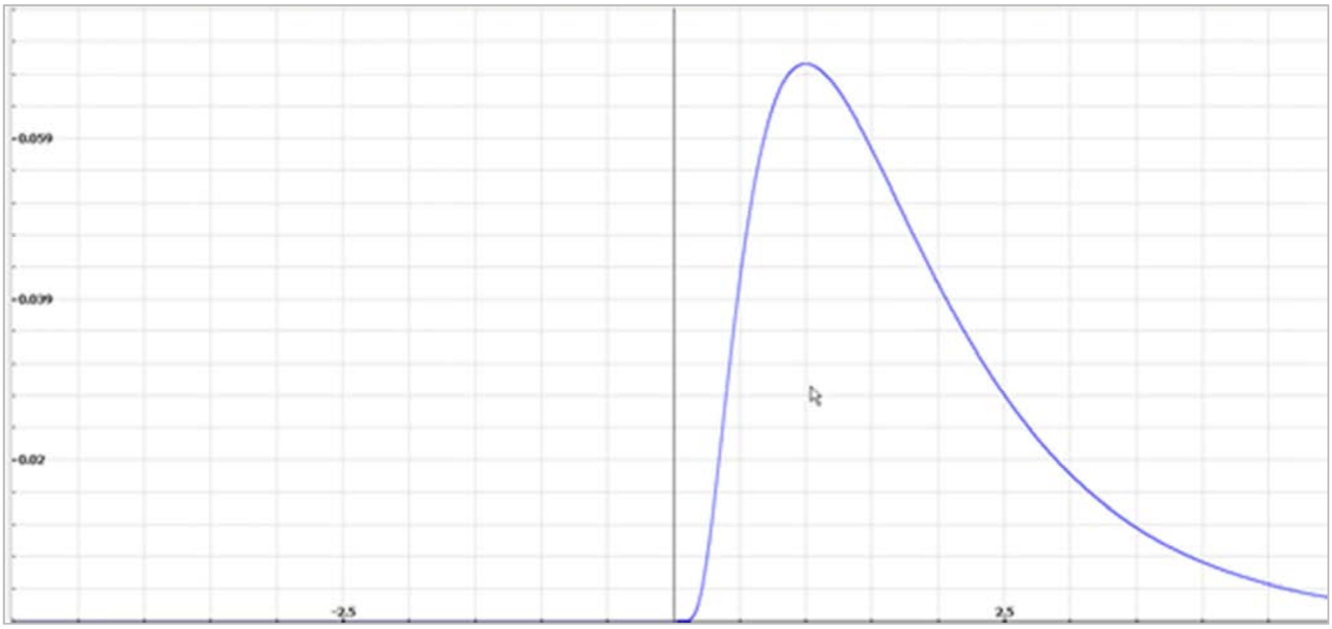


Figure 12. *The efficacy is asymptotically stable to short time intervals.*

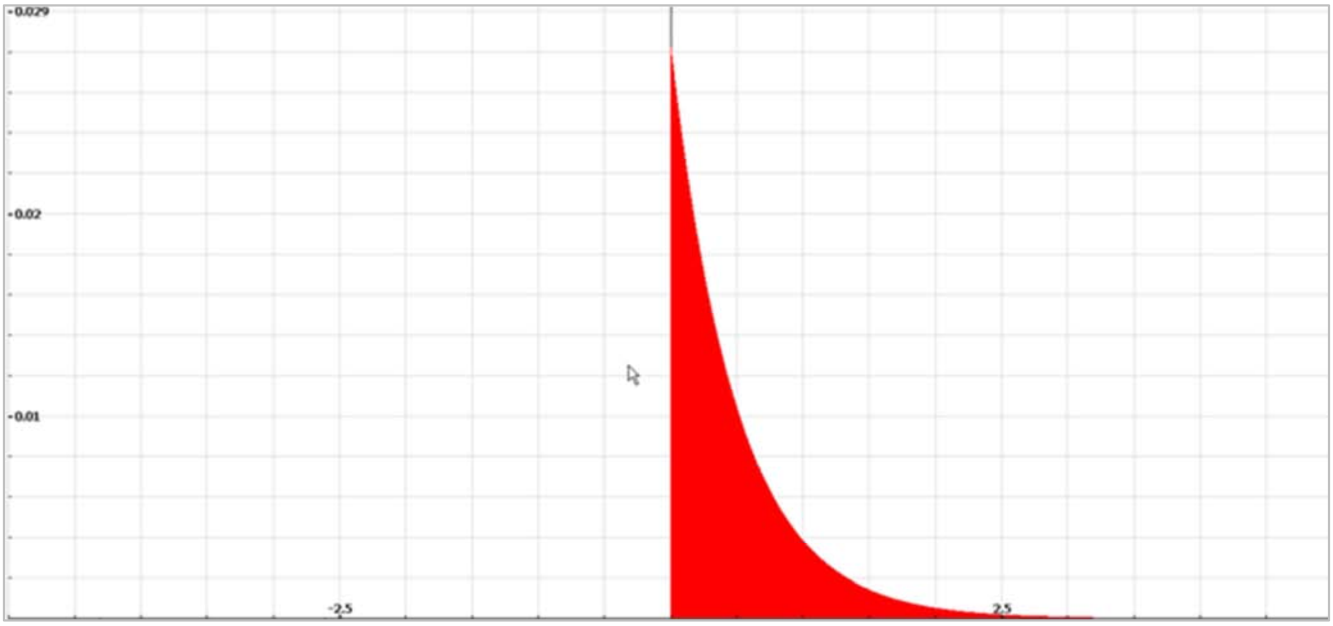


Figure 13. *The efficacy given by (13). The function considered was* $\lambda(x) = (1/36)\exp\{-2x\}u(x).$

LABORATORY CURVES
RESISTIVE EFFECT [Ω] TO DIFFERENT DISTANCE BETWEEN
LIGHT SOURCE AND SENSOR/LDR

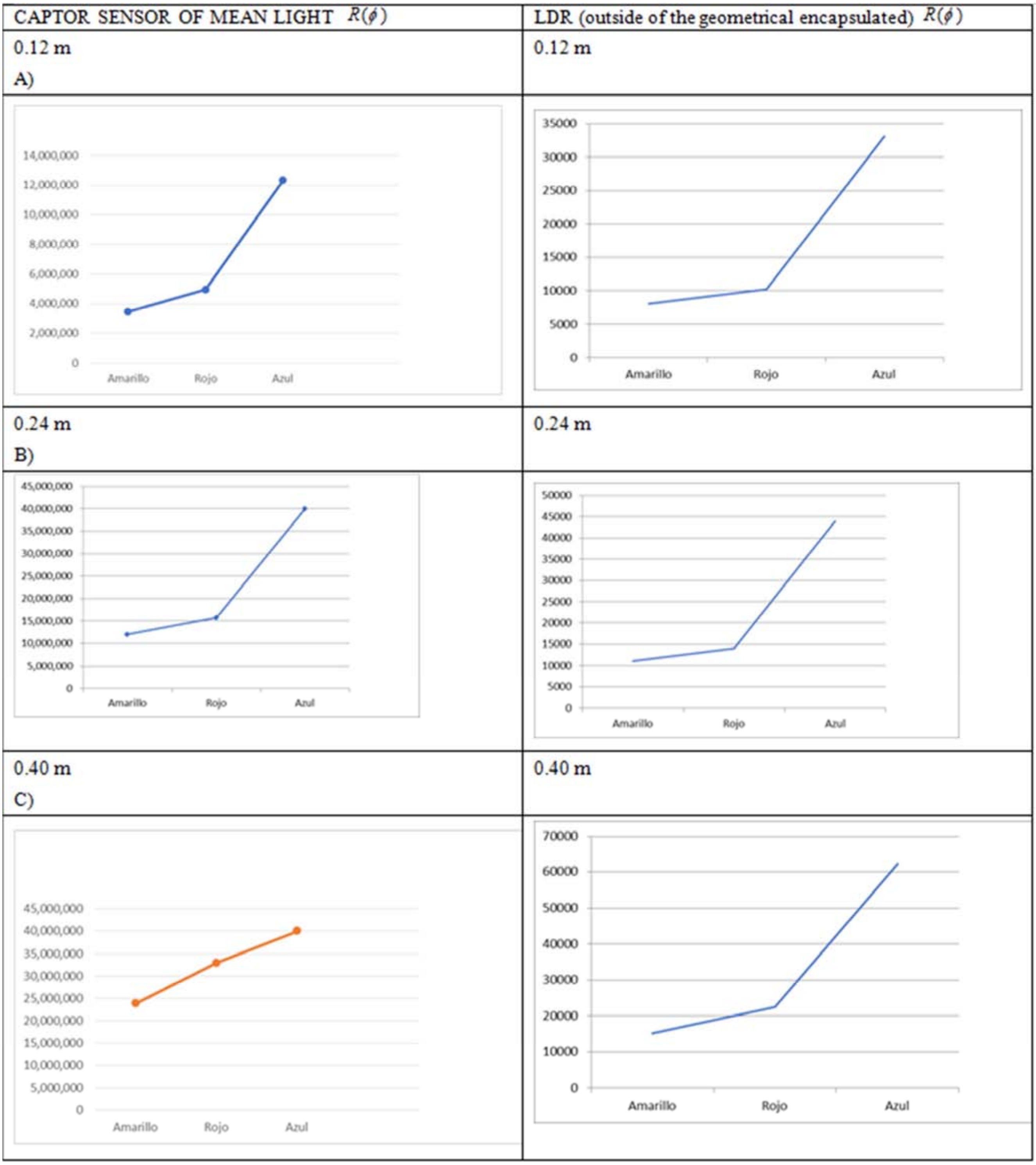


Figure 14. *Comparison photo-resistive response* $R(\varphi)$ *of the captor Sensor of mean light and LDR A). To distance 0.12m. B). To distance 0.24 m. C). To distance 0.40m.*

4. Conclusion

The illuminating sensor designed and created obeys to geometrical principles, which establish their functioning in an optimal way through their mean curvature energy in a dynamical system that consider response signal average expressed by their integral equation. This last, evaluated through the efficacy and efficiency of the system in continuous functioning to different light resonances and size of the space to illuminate (see the Figure 12, Figure 13 and Figure 14). The proposed sensor is so only an example of the several applications of the our concept of curvature energy as field observable in the sensor theory context and the frontier with the microscopic field theory to establish more results in spectral analysis that can useful in the design and development of other advanced electronic devices. The theorem 2. 3. 1, is an example of the spectral analysis results that establish a general way to obtain an analytic efficiency expression to be evaluated considering efficacies until of certain order. The efficiency is 2-dimensional function whose active parameters are the resistance and resonance to sensors based in a photo-resistive component. The Andy's surface is a geometrical model that determines the global behavior of this function.

Acknowledgements

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Technical Notation

$K-$ Curvature as general concept of roundness property. Also used in the paper as Gaussian curvature in a point

$\lambda-$Non-dimensional value of efficacy in a particular time t_0.

$H(\omega_1,\omega_2)-$ 2 – Dimensional spectral mean curvature.

$\lambda(t)-$Efficiency to large time interval. In addition, this function is the curve of energy required to the sensing efficacy. This function is solution of the integral equation to describe the mean behavior of the illuminating sensor.

$V-$ Voltage.

$\Lambda(\omega)-$ Efficacy to a resonance ω.

$A-$ Area.

$R(\varphi)-$Photo-Resistance due to the light flux φ .

$\kappa-$Curvature as value.

$h(t)-$Response signal of the sensor.

$\Lambda(p)-$ Transitory state of the efficiency function $\forall \operatorname{Re} p > -\dfrac{R(\varphi)+1}{R(\varphi)}$.

$\xi-$Illuminating energy density.

$\Xi(\omega)-$Spectra of illuminating energy density.

κ_G-Gaussian curvature in the sense of the value of their integral.

E_v-Luminous energy.

$\kappa(p,\varphi)-$Spectral curvature in the Radon space.

$\mathcal{L}$ – Laplace Operator of Laplace transform.

$\kappa(\omega_1,\omega_2)-$Spectral curvature in the Fourier space.

$V_{in}-$ Input voltage.

LDR-

MLCGD-Mean Light Captor Geometrical Device.

$u(t)-$ Heaviside function.

References

[1] F. Bulnes, I. Martínez, R. Cayetano, I. Valencia, B. Martínez, J. Beltrán, "Curvature as Obstacle to a Photo-Resistor Sensor of Illumining and their Minimal Sensing Region," *Sensors &Transducers Journal*, IFSA, Europe, Vol. 224, No. 2018.

[2] R. A. Adams; Sobolev spaces, Academic Press, New York - San Francisco - London, 1975.

[3] Bulnes, F. Martínez, I, Mendoza, A. Landa, M., "Design and Development of an Electronic Sensor to Detect and Measure Curvature of Spaces Using Curvature Energy," Journal of Sensor Technology, 2012, 2, pp116-126. http://dx.doi.org/10.4236/jst.2012.23017.

[4] Murphy, T. W. (2012). "Maximum spectral luminous efficacy of white light". Journal of Applied Physics. 111 (10): 104909.

[5] Kolmogorov, A. N. and Fomin, S. V. (1975). Elementos de´la Teoría de Funciones y del Análisis Funcional, Mir Moscú.

[6] W. Rudin, Functional Analysis, McGraw-Hill, Inc., USA, 1973.

[7] A. T. Alastalo, V. Kaajakari, "Intermodulation in Capacitively Coupled Microelectromechanical Filters," IEEE Electron Device Letters, Vol. 26, 2005, pp189-191.

[8] Bulnes, F. (1998). *Treatise of Superior Mathematics: Signal and Systems Analysis*, Science Faculty, UNAM-ETC, Mexico.

[9] Bulnes, F. Martínez, I, Mendoza, A. Landa, M., "Design and Development of an Electronic Sensor to Detect and Measure Curvature of Spaces Using Curvature Energy," Journal of Sensor Technology, 2012, 2, pp116-126. http://dx.doi.org/10.4236/jst.2012.23017.

[10] Heath, S. (2003). Embedded systems design. EDN series for design engineers (2ed.)., Elsevier Science, Oxford, UK.

[11] B. Nashelky, "Electronics: Circuit Theory and Electronic Devices," Pearson Prentice Hall, Upper Saddle River, 2003.

[12] Diffenderfes, Robert (2005). Electronic Devices: System and Applications. New Delhi: Delimar.

[13] M. A. Parker, Physics of Optoelectronics, Taylor and Francis, 2004.

[14] M. Bass (ed.), Handbook of Optics Volume I - Devices, Measurements and Properties, 2nd Ed., McGraw-Hill 1995.

[15] M. Bass (ed.), Handbook of Optics Volume II - Devices, Measurements and Properties, 2nd Ed., McGraw-Hill 1995.